Embracing Frustration: Allowing Students to Persevere as Problem Solvers

Teaching Integrated Math and Science Project
University of Illinois at Chicago

www.mathtrailblazers.uic.edu
Embracing Frustration: Allowing Students to Persevere as Problem Solvers

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
perseverance

/ˈpɜrsəvərəns/

noun

noun: perseverance

steadfastness in doing something despite difficulty or delay in achieving success. "his perseverance with the technique illustrates his single-mindedness"

synonyms: persistence, tenacity, determination, staying power, indefatigability, steadfastness, purposefulness; patience, endurance, application, diligence, dedication, commitment, doggedness, assiduity, tirelessness, stamina; intransigence, obstinacy; informal stick-to-it-leness; formal pertinacity

"in a competitive environment, perseverance is an invaluable asset"
Research Studies

• Whole Number Study–UIC & KSU  2003–2008
• Implementation Study–UIC  2003–2006
• Fractions and Ratios–UMN  2004–2006
• Video Study–UIC  2003–2006
• Field Test Study–UIC  2006–2010
• Student Achievement Study–UIC  2009–2011
• Embedded Assessment Study–UIC  Current
Workshop Goals

- Learn how to guide students as they
  • engage in high-cognitive demand activities,
  • communicate strategic thinking,
  • critique problem-solving strategies,
  • and use tools and multiple strategies to solve rich problems.

- Learn how formative embedded assessment and providing meaningful feedback moves learning forward.
Mathematics Teaching Practices

• Establish mathematics goals to focus learning.
• Implement tasks that promote reasoning and problem solving.
• Use and connect mathematical representations.
• Facilitate meaningful mathematical discourse.
• Pose purposeful questions.
• Build procedural fluency from conceptual understanding.
• **Support productive struggle in learning mathematics.**
• Elicit and use evidence of student thinking.

-NCTM’s *Principles to Action*
Support productive struggle in learning mathematics.

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

- NCTM Principles to Actions

Students’ struggles are opportunities for diving more deeply into understanding the math instead of simply seeking correct solutions.
Feeling Good about the Struggle
A Classroom Culture that Allows Students to Persevere in Problem Solving...

✓ Embodies problem-based learning
✓ Allows time for students to reason mathematically
✓ Provides opportunities for students to freely discuss mathematical understandings
✓ Requires that students explain how they solve problems
✓ Lets students be responsible for their own learning
✓ Sets clear expectations
✓ Provides frequent formative assessment and feedback
✓ Teaches students to work like real mathematicians
✓ Creates a flexible and dynamic classroom environment
✓ Allows opportunities for further exploration of a topic

- Pualee, Seitz, & Hollingshead 2013
Productive or Unproductive Beliefs?

① An effective teacher makes the mathematics easy for students by guiding them step-by-step through problem-solving to ensure that they are not frustrated or confused.

② An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics. (Productive)
Productive or Unproductive Beliefs?

① The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.

② The role of the teacher is to engage students in tasks that promote reasoning and problem-solving and facilitate discourse that moves students towards shared understand of mathematics. (Productive)
Productive or Unproductive Beliefs?

① The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others. (Productive)

② The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.
Creating a Safe Place for Problem Solving

Teachers can facilitate this by:

• providing time for student exploration and inquiry, and allowing students to grapple with problems

• carefully selecting problems to discuss, including problems where there are often misconceptions

• respectfully reacting to incorrect answers and communicating that these errors provide important learning opportunities
Mrs. Dewey has $20 to buy lunch for herself and three friends. Burritos at Burrito Barn cost $4.12. Is $20 enough for four hungry teachers to buy burritos? If so, will there be any money left over to buy sodas? How much money?

-Adapted from Jen Saul’s 3rd Grade Class
Find Three Ways

Find three ways video
https://www.teachingchannel.org/videos/problem-solving-math
What Does it Look Like?

<table>
<thead>
<tr>
<th>Support productive struggle in learning mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What are teachers doing?</strong></td>
</tr>
<tr>
<td><strong>What are students doing?</strong></td>
</tr>
</tbody>
</table>
Implement tasks that promote reasoning and problem solving.

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving and allow multiple entry points and varied solution strategies.

- NCTM Principles to Actions
Productive or Unproductive Beliefs?

① Mathematics learning should focus on developing understanding of concepts and procedures through problem-solving, reasoning, and discourse. (Productive)

② Mathematics learning should focus on practicing procedures and memorizing basic number combinations.
High-Cognitive Tasks

Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature.

-(Boaler & Staples ‘08; Hiebart & Wearne ‘93; Stein & Lane ‘96)
What are High-Cognitive Tasks?

✓ Build mathematical proficiency

✓ Challenge and enrich student thinking

✓ Develop mathematical vocabulary related to the ideas and concepts

✓ Promote the ability to learn multiple ways of thinking about and representing mathematical ideas (pictures, diagrams, concrete models, numeric/symbolic)

-NCSM Great Tasks for Mathematics K-5
(Shrock, Norris, Pualee, Seitz, & Hollingshead) 2013
A great task:

- Revolves around an **interesting problem** that offers several methods of solution.

- Is directed at **essential mathematical content** as specified in the standards.

- Is **challenging**, requiring examination and perseverance.

- Provides for rich **discourse** on the mathematics involved.

- Builds student understanding-with a clear set of learning **expectations**.

- Includes opportunity for **reflection** and extension.
Carlos’ Thinking

Choose 3 Ways!

Circle the ways you would like to show your thinking.

- Dollar a Picture
- Skip Count
- Repeated Addition
- Multiple
- Divide
- Make a Table

What do you know?
- I know that each burrito costs $4.12.
- There is going to be 4 teachers.

What do you need to know?
- Could you buy every teacher a burrito with $20.00?

What is the answer?
- The answer I got was $16.48.
### Multiplication Strategies Menu

#### Break into tens and ones

**Using Expanded Form**

\[
\begin{align*}
23 & = 20 + 3 \\
& \quad \times 6 \\
& = 120 + 18 = 138 \\
\end{align*}
\]

or

\[
\begin{align*}
20 & \quad 3 \\
6 & \quad 20 = 120 \\
& \quad 6 \times 3 = 18 \\
& \quad \frac{120 + 18}{138}
\end{align*}
\]

**Using All-Partials**

\[
\begin{align*}
23 & \quad 23 \\
& \quad \times 6 \quad \times 6 \\
18 & \quad 120 \quad + 18 \quad \frac{120 + 18}{138}
\end{align*}
\]

**Compact Method**

\[
\begin{align*}
\frac{23}{138}
\end{align*}
\]

#### Other ways to use simpler problems

**Thinking About Money**

\[
\begin{align*}
27 \times 4 & = 25 \times 4 + 2 \times 4 \\
& = 100 + 8 \\
& = 108
\end{align*}
\]

**Using Simpler Numbers**

I know \(48 + 2 = 50\).
So, \(50 \times 6 = 300\) and \(2 \times 6 = 12\).
Then I subtracted \(300 - 12 = 288\).

**Another Strategy:** ____________________________
Is it a Great Task?

Counting Squares

1. How many small squares are in the large rectangle below? Show or tell how you found your answer.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

- NCTM Principles to Actions

Chris’s answer makes sense. 21 hundredths divided by 3 hundredths is 7.
To Facilitate Meaningful Discourse:

- try to **anticipate** the strategies students will use in solving a problem
- **monitor** students’ work as they solve problems
- **select** students’ strategies to be shared rather than choosing random students
- **organize students’ sharing** so that it will be helpful to the class discussion
- encourage students to **make connections** between a variety of strategies so that they see the math behind them is the same
Creating a Safe Place for Problem Solving

Teachers can facilitate this by:

- providing time for student exploration and inquiry, and allowing students to grapple with problems
- carefully selecting problems to discuss, including problems where there are often misconceptions
- respectfully reacting to incorrect answers and communicating that these errors provide important learning opportunities
Counting Squares

1. How many small squares are in the large rectangle below? Show or tell how you found your answer.

---

Counting Squares

From the Fish Hatchery

SAB • Grade 4 • Unit 11 • Lesson 1
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.

- NCTM *Principles to Actions*
Shannon’s Strategy

\[
\begin{array}{c}
140 \\
28 \\
28 \\
28 \\
28 \\
\hline \\
140 \\
28 \\
28 \\
28 \\
28 \\
\hline \\
252
\end{array}
\]
Nila’s Strategy

\[
\begin{array}{c}
14 \\
\times 18 \\
\hline
100 \\
80 \\
40 \\
+ 32 \\
\hline
252 \\
\end{array}
\]
Five Effective Talk Strategies

1. **Revoicing:** (“So, you are saying that it’s an odd number?”)

2. **Repeating:** Asking a student to restate someone else’s thinking. (“Can you repeat what he just said using your own words?”)

3. **Reasoning:** Asking students to respond to the someone else’s reasoning. (“Do you agree or disagree? Explain your reasoning.”)

4. **Adding on:** Prompting students for further participation. (“Would someone like to add something more to this?”)

5. **Waiting:** Using wait time. (“Take your time…Gather your thoughts…”)

-Classroom Discussions: Using Math Talk to Help Student Learn
Discuss:

• rules for listening
• ways to agree and disagree respectfully
• how to critique the reasoning of others rather than the person
• personal responsibilities
• how to take turns
• ways to support facts or opinions
Elicit and use evidence of student thinking.

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend.

- NCTM *Principles to Actions*
## Fraction Puzzles

### Assessment Puzzle Clues

<table>
<thead>
<tr>
<th>PUZZLE D</th>
<th>PUZZLE D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make a shape with the value of $\frac{5}{6}$.</td>
<td>The red circle equals 1 whole.</td>
</tr>
<tr>
<td>Use 3 or 4 pieces.</td>
<td>Use at least 1 aqua piece, but not all aquas.</td>
</tr>
<tr>
<td></td>
<td>Use no blue pieces.</td>
</tr>
</tbody>
</table>

Mines says, "The red circle is equal to one whole."
Some Possible Solutions

\[
\frac{1}{2} + \frac{2}{6} = \frac{5}{6}
\]

\[
\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{5}{6}
\]

\[
\frac{2}{3} + \frac{1}{6} = \frac{5}{6}
\]

\[
\frac{2}{4} + \frac{2}{6} = \frac{5}{6}
\]
Feedback Boxes

We placed $\frac{1}{6}$ on the red circle (1 whole) because that was the leftover. Then we built around it with other fractions to make the red circle or 1. Then we checked to be sure that it fit the clues.

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{4} + \frac{1}{4} = \frac{5}{6}$$

<table>
<thead>
<tr>
<th>MPE3. Check for reasonableness. I look back at my solution to see if my answer makes sense. If it does not, I try again.</th>
<th>Yes...</th>
<th>Jerome said he did, but he didn’t show how his solution matched each step.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPE5. Show my work. I show or tell how I arrived at my answer so someone else can understand my thinking.</td>
<td>Yes, but...</td>
<td>He told that he started with an aqua piece for $\frac{1}{6}$ and put fractions around it for $\frac{5}{6}$. But, he did not show or tell what pieces he used or if he traded any.</td>
</tr>
</tbody>
</table>
15. Roberto is making Trail Mix. He has a $\frac{1}{8}$-cup measuring scoop and a 1-cup measuring scoop. His recipe is below.

Explain how Roberto can use the measuring scoops he has to mix the recipe correctly.

Trail Mix
Mix together:
• 1$\frac{7}{8}$ cups peanuts
• 1$\frac{1}{2}$ cups raisins
• $\frac{3}{4}$ cup chocolate chips
Makes 6 servings.

Workshop: More Than, Less Than, or Equal To
Check-In: Q# 15 Feedback Box

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Check-In</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find equivalent fractions</td>
<td>E8</td>
<td></td>
</tr>
<tr>
<td>• Using models</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Using multiplication and division strategies</td>
<td>E8</td>
<td></td>
</tr>
<tr>
<td>Compare and order fractions using area models</td>
<td>E9</td>
<td></td>
</tr>
</tbody>
</table>

MP.E1. Find a strategy. I choose good tools and an efficient strategy for solving the problem.

Yes . . . Yes, but . . . No, but . . . No . . .

MP.E5. Show my work. I show or tell how I arrived at my answer so someone else can understand my thinking.
Exit Slip

You hand out a slip of paper with a few simple questions, and students give them to you as they leave your class.

<table>
<thead>
<tr>
<th></th>
<th>Things I Learned Today...</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Things I Found Interesting...</td>
</tr>
<tr>
<td>1</td>
<td>Question I Still Have...</td>
</tr>
</tbody>
</table>
Praise and Mindsets

YOU MUST BE SMART AT THIS

YOU MUST HAVE WORKED HARD

intelligence
effort
Creating a Safe Place for Problem Solving

Teachers can facilitate this by:

• providing time for student exploration and inquiry, and allowing students to grapple with problems

• carefully selecting problems to discuss, including problems where there are often misconceptions

• respectfully reacting to incorrect answers and communicating that these errors provide important learning opportunities
I 💙 Mistakes

“Tell students that you love mistakes and that they will be valued at all times. Tell them that it is good to make mistakes as we know that when people make mistakes, their brains are growing. This single message can be incredibly liberating for students.”

-Jo Boaler
My Favorite No: Learning From Mistakes

My Favorite No: Learning From Mistakes -
https://www.teachingchannel.org/videos/class-warm-up-routine
What Does it Look Like?

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But I want to rescue them!

Although well-intended, rescuing students will
• undermine students’ efforts
• lower the cognitive demand of the task,
• deprive students of opportunities to engage fully in sense-making of mathematics

-Reinhart 2000; Stein et al. 2009
Jacob has 28 square carpet tiles. He wants to put them together to make a rectangular rug to cover part of the den floor. He tries several arrangements. He seems to like the rug that has 4 rows. How many tiles are in each row? How do you know?

What other ways can Jacob arrange 28 square carpet tiles into a rectangle?

Is 28 a multiple of 4? How do you know?

Is 28 a multiple of 3? How do you know?

Is 5 a factor of 28? How do you know?
TIMS Candy Company Box

Design a box for the TIMS Candy Company that will hold 36 pieces of candy and has more than two layers. Each layer must have the same number of pieces. Tell how many layers are in your box. Also, tell how many pieces of candy are in each layer.

Show or tell how you solved this problem.
Tools to Support Problem Solving
Design a box for the TIMS Candy Company that will hold 36 pieces of candy and has more than two layers. Each layer must have the same number of pieces. Tell how many layers are in your box. Also, tell how many pieces of candy are in each layer.

Show or tell how you solved this problem.
Status Check

36 + 36 = 72
## Status Check

<table>
<thead>
<tr>
<th>Rows</th>
<th>Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
Support without Rescuing

✓ Stop working
✓ Write down two things you know about the problem
✓ Write down one thing you wish you knew
✓ Discuss as a class what to do next
✓ Consider the various ideas as you continue to work on the task
Justify and Critique

One person explains how they solved the problem to the other person. Then discuss together:

• Does your partner’s answer make sense? Why or why not?
• Can you explain your partner’s solution in your own words?
• What is the same about your solutions?
• What is different?
# What Does it Look Like?

**Support productive struggle in learning mathematics**

<table>
<thead>
<tr>
<th>What are teachers doing?</th>
<th>What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Anticipating where students might struggle &amp; being prepared to support</td>
<td>• Struggling at times but knowing that is part of learning</td>
</tr>
<tr>
<td>• Giving time to struggle</td>
<td>• Asking and answering questions to clarify thinking and understanding</td>
</tr>
<tr>
<td>• Posing questions that scaffold and reengage student thinking without rescuing</td>
<td>• Realizing it is ok to ask for help but not to give up</td>
</tr>
<tr>
<td>• Providing feedback and facilitating discussions to deepen understanding</td>
<td>• Supporting classmates by sharing strategies, justifying solutions, and critiquing reasoning</td>
</tr>
<tr>
<td>• Communicating that struggle and mistakes are a valuable part of learning</td>
<td></td>
</tr>
<tr>
<td>• Praising students for their effort and perseverance</td>
<td></td>
</tr>
</tbody>
</table>

Struggling at times but knowing that is part of learning
Why embrace frustration?

“Teaching that embraces and uses productive struggle leads to long-term benefits, with students able to apply their learning to new problem situations.”

– Kapur 2010
Embracing Frustration: Allowing Students to Persevere as Problem Solvers

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Websites

Dr. Jo Boaler - Four Boosting Math Messages from Jo and Her Students
https://www.youcubed.org/students/

Dr. Jo Boaler - Growth Mindset
https://www.youcubed.org/category/teaching-ideas/growing-mindset/

Persistence in Problem Solving - Find Three Ways
https://www.teachingchannel.org/videos/problem-solving-math

Dr. Carol Dweck - A Study on Praise and Mindsets
https://www.youtube.com/watch?v=NWv1VdDeoRY

David Wees' 56 Different Examples of Formative Assessment
http://davidwees.com/content/formative-assessment/

My Favorite No: Learning From Mistakes -
https://www.teachingchannel.org/videos/class-warm-up-routine

Dr. Carol Dweck - On Struggle
https://www.teachingchannel.org/videos/embracing-struggle-exl

Standards for Mathematical Practice: Commentary and Elaborations

NCTM Common Core Videos -